Assuming that the space velocity of IC711 relative to the cluster medium is σ_T, the angle between the velocity vector of this galaxy and our line of sight is 70°. In this case the radio tail is ~900 kpc long. Using these values for the galaxy velocity and tail length with Vallee and Wilson's estimate of the magnetic field ($H = 4\times10^{-6}$ gauss) we may estimate the importance of *in situ* acceleration for this radio tail.

From the equations for energy loss due to inverse Compton and synchrotron emission[3] we calculate the e-folding decay time, t, for the energy of particles in the tail. Dividing the tail's length by the estimated velocity of the galaxies with respect to the centre of the cluster gives T, the length of time that particles emit detectable energy[1]. Then the ratio $N = T/t$ is a measure of the effect of *in situ* particle acceleration in the radio tail. For IC711 we find $N\simeq20$, a typical value from the Pacholczyk and Scott[2] theory. For IC708 a similar calculation yields $N\simeq4$.

From our spectroscopic study of galaxies in Abell 1314 we conclude that both reported radio tail galaxies are cluster members, and that *in situ* acceleration (with a magnitude consistent with Pacholczyk and Scott's[2] model) is an important effect in their radio components.

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White dwarfs, the Galaxy and Dirac's cosmology

AN apparent absence, or deficiency, of white dwarfs fainter than $\sim10^{-4}L_\odot$ has been noticed for a long time[1]. Often attributed to incomplete statistics for the coolest, faintest objects[2,3], this deficiency has nevertheless not been removed even after the most persistent spectroscopic surveys by Greenstein[4,5] (see also Liebert[6]). Moreover, Jones's[7] analysis of reduced proper motions for the very faint proper-motion stars observed by Eggen and by Luyten tends only to confirm the deficiency. If real, the lack of very faint white dwarfs may have its origin in a rapid drop of the internal heat content of the white dwarf as a result of ion crystallisation and of envelope convection when the star becomes sufficiently cool (see Greenstein[4] and Van Horn[8] and references therein). But perhaps it will eventually prove to be difficult, as Lamb and Van Horn[9] have already found, to increase the cooling rate by a sufficiently large amount to account for all the deficiency. Another explanation is proposed here on the basis of Dirac's[10–12] cosmological hypothesis that the gravitational constant, G, varies with the elapsed time since the beginning of the expansion of the Universe as t^{-1} and the number of particles in the Universe increases as t^2 (if the measurements are made in atomic units). For a white dwarf, the Chandrasekhar mass limit is a collection of fundamental constants proportional to $G^{-3/2}$, and therefore increases with time as $t^{3/2}$. In the 'additive' version of Dirac's theory, the actual mass, M, of a relatively small object like a star remains essentially unchanged by the creation of new matter in the Universe, and so a white dwarf becomes 'more stable' as time goes on. But, in the 'multiplicative' version of his theory, M increases as t^2 and may eventually exceed the Chandrasekhar limit. If so, gravitational collapse of the white dwarf into an invisible black hole or neutron star will quickly ensue. Since some possible empirical support has already been adduced for the multiplicative theory, although not for the additive theory[13–17], it is interesting to determine whether the multiplicative theory may also have something to do with the apparent deficiency of faint white dwarfs and whether there are any possible consequences for galactic evolution.

First it is necessary to evaluate what would be the luminosity of a white dwarf (assumed to be in hydrostatic equilibrium) if the formal change of internal energy and gravitational potential energy of the star arising from the creation of new matter is available as free energy partly to raise the heat content of the star and partly to be radiated away at the surface, and is the sole source of visible luminosity. The changes of G and M are assumed to be given by

$$G(t) = G_0[t/t_0]^{-1} \tag{1}$$

and

$$M(t) = M_0[t/t_0]^2 \tag{2}$$

where the subscript 0 refers to the present epoch. Although these expressions give an undesirable zero mass and infinite G at $t = 0$, they are only needed at comparatively large fractions of t_0, at which times they may serve as suitable approximations for the present purpose (Van Flandern's[16] observed value for the residual secular acceleration of the Moon, interpreted as being carried by a decrease of G, is consistent with equation (1) for $t_0 = 12\times10^9$ yr). The mass–radius relationship[18] relevant to most of the appropriate range of white-dwarf masses is approximately

$$\left[\frac{G}{G_0}\right]^2\frac{R}{R_0}\simeq\left[\frac{M}{M_0}\right]^{-1} \tag{3}$$

The luminosity follows from evaluating the rate of change of total (internal plus potential) energy

$$L = \frac{d}{dt}\left[\beta\frac{GM^2}{R}\right] \tag{4}$$

where β extends from 3/7 to 0, for masses ranging from very low values to values close to the Chandrasekhar limit, respectively; β may, however, be held constant here. The result is, at the present epoch

$$L = 3\beta GM^2/Rt_0 \tag{5}$$

For a typical white dwarf with $M = 0.8\,M_\odot$, $R = 10^{-2}R_\odot$, and $\beta = 0.4$, and for an approximate age for the Universe of $t_0 = 12\times10^9$ yr (refs 16, 19), the expected luminosity is $L = 0.2\,L_\odot$. (An analogous derivation for main-sequence stars shows that the 'extra' luminosity is insignificant, except for the faintest main-sequence stars; in general, it will be negligible for any star whose Helmholtz–Kelvin time, GM^2/RL, is short compared with t_0.)

Since so high a white-dwarf luminosity is in serious conflict with observations[2], it may be concluded either that Dirac's theory is untenable, or that the radiation (and 'heating') is of an unknown type, or that the process of creation of new matter requires a corresponding supply of energy. For lack of a better alternative, I shall simply assume that the 'extra' energy somehow goes into work done creating the new matter and that the ordinary rates of stellar energy generation should be used to determine stellar temperatures and luminosities. Therefore, at least in its macroscopic effect, the process of creation is assumed to be a collective phenomenon involving the whole mass of the star; but this may be an unavoidable requirement of the multiplicative theory anyway, since Dirac[12] has already pointed out that, in the case of the Earth, new atoms may have to form at the surfaces, rather than in the interiors, of rock crystals. In the case of gaseous stars, I shall follow Dirac in assuming that new atoms emerge with the same basic properties, that is, species, state, and general location, as the existing atoms.

To proceed further, simple physical assumptions about the structure and luminosity source of white dwarfs will be adopted here: (1) Chandrasekhar's[18] classic mass limit of $1.44M_\odot$ for a white dwarf at the present epoch; (2) Mestel's[20] simplified cooling theory of white dwarfs, as presented by Schwarzschild[21]; and (3) an average mass of $0.8M_\odot$ for young (recently formed) white dwarfs. How this average mass has varied in the past is not easy to determine, but a good assumption is that it has been proportional to $G^{-3/2}$, since most critical stellar masses in astrophysics, such as the approximate upper and lower mass limits of ordinary stars[22,23], have this proportionality.

Schwarzschild's[21] standard relationship between the luminosity, L, of a white dwarf and its isothermal core temperature, T, is

$$L = KGMT^{3.5} \qquad (6)$$

where K is a constant depending on the chemical composition of the radiative envelope. The luminosity of the white dwarf is supposed to be supplied solely by thermal cooling of the non-degenerate ions; the basic equation of thermal equilibrium is given by

$$\frac{\mathrm{d}L(r)}{\mathrm{d}M(r)} = -\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{3}{2}\mathscr{R}T\right] \qquad (7)$$

whose correct integrated form is

$$L = -\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{3}{2}\mathscr{R}T\right]M \qquad (8)$$

Solution of equations (1), (2), (6) and (8) for the core temperature can be simplified by assuming that the initial temperature is 'infinitely' hotter than the present temperature. Then the time, t_f, at which the white dwarf actually formed is found to be given by

$$t_f/t_0 = \exp(-t_{cl}/t_0) \qquad (9)$$

where

$$t_{cl} = \left[\frac{3}{2}\mathscr{R}T\right]/(2.5\,L) \qquad (10)$$

and all quantities in parentheses are evaluated at the present epoch. Note that t_{cl} is simply the classical cooling time that would apply if G and M remained constant in time. For the brighter luminosities characteristic of most of the observed white dwarfs, the revised cooling times are not much different from the classical cooling times and therefore do not destroy the approximate agreement with observations already achieved[2].

The most recent period at which a white dwarf with mass typical of its epoch could have formed to reach the Chandrasekhar mass limit at the present epoch follows from $0.8(t_f/t_0)^{3/2} = 1.44(t_f/t_0)^2$ and is $t_f = 0.31t_0$. Substitution of this result into equation (9) yields $t_{cl} = 1.2t_0$, which may be directly entered into Schwarzschild's[21] Table 27.1 for the classical cooling times of white dwarfs as a function of their luminosities, if his cooling times are multiplied by a factor 1.3 to correspond to a mass of $1.44\,M_\odot$ and by another factor 0.3 to convert from a predominantly helium core composition to a more probable carbon core composition. With $t_0 = 12\times 10^9$ yr, a value of $L = 2\times 10^{-5}L_\odot$ is found for the present luminosity of an 'average' white dwarf that has cooled for $t_0 - t_f = 8\times 10^9$ yr. Older white dwarfs would long ago have faded into invisibility through gravitational collapse. (Some older white dwarfs of smaller-than-average initial mass would still be visible, of course.) Most younger white dwarfs would not yet have collapsed. It is interesting to note that most of the old white dwarfs with high space velocities[1,24] have large photometrically implied masses. Could some of these objects be on the verge of gravitational collapse? Accurate measurements of the surface gravities of the oldest white dwarfs will be necessary to determine whether these stars are, on the average, heavier than the brighter objects. Noticeable differences are expected to occur only for $L < 10^{-3}L_\odot$.

Allowance for a probable initial spectrum of white-dwarf masses, for the uncertainty of the adopted chemical composition, for known corrections to the classical white-dwarf mass limit, for various inadequacies of the white-dwarf cooling theory used here, and for the uncertainty of the adopted age of the Universe leads only to a general order of magnitude of 10^{-5} to $10^{-4}L_\odot$ as the predicted cutoff luminosity for most white dwarfs. This prediction seems to be in as good agreement with the available observations as could be expected.

The earliest generation of stars in the Galaxy probably formed within one galactic rotation period after the onset of collapse of the protogalaxy[25]. At the present time, this period in the outer galactic regions near the Sun is 2×10^8 yr, but, in Dirac's theory, it must be scaled down as t, since the rotational velocity stays at the same fraction of the speed of light while the radius of the Galaxy changes as t. The first white dwarfs could have originated about one main-sequence lifetime (which also scales as t—see below) after that era. Since the collapse of the protogalaxy could have begun as late as $0.25t_0$ (ref. 19) the foregoing stellar formation time scales are seen to be relatively short in comparison, and the first-generation white dwarfs can therefore be expected to be, at the present time, invisible black holes or neutron stars with typical masses of $>1.6\,M_\odot$. Consequently, simple counts of observed stars in the solar neighbourhood will, if uncorrected for cosmological effects, lead to errors in the extrapolated numbers and masses of dead stars formed since the collapse of the protogalaxy. (The present discussion will ignore the probably small contribution to the mass of the Galaxy from black holes and neutron stars that have formed as direct end-products of normal stellar evolution.)

At first sight, it may seem that a gross underestimate must have been made in the standard extrapolation for dead stars[21]. That this is not so can be seen upon the following reflection. Consider the average properties of typical bright main-sequence stars that give rise to white dwarfs. Their average main-sequence lifetime, t_{MS}, at any epoch t may be regarded as short compared to t (since it is known to be short at the present epoch and it will be shown that t_{MS}/t remains approximately constant in time). Then their birthrate follows simply as $\mathrm{d}N_{MS}/\mathrm{d}t = N_{MS}/t_{MS}$, where N_{MS} is the number of such stars present in the Galaxy at time t. The next task is to evaluate $t_{MS} = qEM/L$ by noting that q, the mass fraction of hydrogen eventually consumed in the star[26], and E, the energy released per gram of material, are independent of t, while the average mass, M, of these bright main-sequence stars at the time of their formation (like all critical stellar masses) probably varies as $G^{-3/2} \propto t^{3/2}$ and their luminosity, L, scales as G^4M^3 (electron-scattering opacity) or as G^7M^5 (Kramers opacity). In either case $L \propto t^{1/2}$. Certainly the evolutionary change in L/M can be neglected, both because $t_{MS} \ll t$ and because this change will in any case be small ($G \propto t^{-1}$, $M \propto t^2$, and hence $L/M \simeq$ constant). It then follows that $t_{MS} \propto t$. Next, the crude assumption is made that the ratio of the total mass in the form of bright main-sequence stars to that of interstellar gas in the Galaxy remains approximately constant after the collapse of the protogalaxy (anticipating the final result that the permanent loss of gas now tied up in dead stars is small). It then follows that $N_{MS} \propto t^{1/2}$, since the mass of the average bright star when it forms varies with the epoch as $t^{3/2}$ whereas the total mass of interstellar gas in the Galaxy increases as t^2. The birthrate of bright stars becomes $\mathrm{d}N_{MS}/\mathrm{d}t \propto t^{-1/2}$.

The deathrate of these stars may be assumed to be equal to their birthrate, so that the rate of addition of mass in the form of new white dwarfs to the Galaxy is $0.8M_\odot(t/t_0)^{3/2}$ times

$(dN_{MS}/dt)_0(t/t_0)^{-1/2}$. The rate of growth of the masses of dead stars that have already formed follows from equation (2) as $2M_{dead}(t)/t$. The sum of these two terms yields the total rate

$$\frac{dM_{dead}(t)}{dt} = \frac{2M_{dead}(t)}{t} + 0.8M_\odot\left[\frac{dN_{MS}}{dt}\right]_0\frac{t}{t_0} \quad (11)$$

By setting $M_{dead}(t) = 0$ at $t = t_i$ (corresponding to the end of the collapse of the protogalaxy) and by noting that the quantity

$$M_{dead}{}^* = 0.8M_\odot(dN_{MS}/dt)_0 t_0 \quad (12)$$

is the result of the standard extrapolation for dead stars, the solution of equation (11) is

$$M_{dead}(t) = M_{dead}{}^*[t/t_0]^2\ln(t/t_i) \quad (13)$$

At the present time, $M_{dead} = M_{dead}{}^*\ln(t_0/t_i)$. Dead stars that formed at any time earlier than $t_f = 0.31t_0$ will now, however, be invisible objects rather than white dwarfs. If the total mass of dead stars at time t_f is multiplied by $(t_0/t_f)^2$, it follows that the present total mass of collapsed objects alone is $M_{collapsed} = M_{dead}\ln(t_f/t_i)$. Further, the present total mass of white dwarfs alone is found to be $M_{WD} = M_{dead}{}^*\ln(t_0/t_f)$, which is not significantly different from $M_{dead}{}^*$!

With reasonable choices for the collapse epoch, say $t_i/t_0 = 0.25-0.01$, the sought-for result is finally obtained: $M_{dead}/M_{dead}{}^* = 2$–$5$ and $M_{collapsed}/M_{WD} = 0.2$–$3$. These are modest values. Therefore, in the framework of Dirac's multiplicative theory, there is no special mechanism to produce the vast numbers of black holes or other dead stars that are sometimes suggested to compose the 'missing matter' in the solar neighbourhood and in the galactic halo.

The predicted paucity of stars during the past history of the Galaxy arises from the smaller galactic mass then, and implies that the Milky Way must have been considerably dimmer than it is at present. Most of the luminosity may be assumed to have come then (as now) from the few brightest stars. Assuming that these stars form with masses in proportion to $G^{-3/2}$ leads, as above, to $L \propto t^{1/2}$ and $N_{brightest} \propto t^{1/2}$, so that the total luminosity of the Galaxy, $N_{brightest}\,L$, is found to rise as t (but its colour probably changes relatively little with time). Thus, at the end of the original collapse phase of the Galaxy, after the initial metals abundance had been produced and supermassive stars were no longer being formed, the Milky Way would have been only 0.01–0.25 times as bright as it is today *ceteris paribus*. It could have been even fainter if the upper end of the allowable range of stellar masses were underpopulated when the galactic mass was small (an effect observed today in external galaxies of small mass). But, if the mass of the Galaxy had not been varying along with G, the Milky Way would have been brighter in proportion to t^{-1} in the past than it is today. Observations of distant spiral galaxies may eventually be able to discriminate between these various evolutionary models.

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Extraction of metals from basalt by humic acids

In many studies, the activity of humic acid in weathering and other geological processes has been considered negligible. Only simple, low molecular weight organic acids are usually considered effective in mineral degradation[1]. Those acids, however, frequently constitute only a minor fraction of the soil organic matter, particularly in semi-arid and arid areas. In Israel, fulvic acid/humic acid ratios in the organic matter of major soil types range, with one exception, from 1.5 to 6 (ref. 2). Humic acids are also well represented in the organic matter of many sediments[3,4]. The effect of these high molecular weight compounds on mineral decomposition and metal dissolution is therefore of great interest in the earth sciences. The results presented here suggest that humic acids also may have a significant role in rock solubilisation and weathering, because of their capacity to extract considerable amounts of metals, particularly copper and zinc, from basalt rock.

Though the part played by fulvic acids and other low molecular weight organic acids in weathering has been widely documented, the action of humic acids in these processes has been examined in only a few cases. Schalscha *et al.*[5] extracted considerable amounts of Fe from minerals treated with humic acids. The solubilisation of major elements from alumosilicates has been shown to be considerably speeded up by treatments with humic acids or organic acid components of humic acid[6–8].

Baker[9] has reported on the strong solvent activity of humic acid from a podzolic soil in Tasmania, towards a number of minerals and metals. In his studies, separate mineral species were exposed to the action of humic acids. Here the effect of humic acids on basalt rock has been examined with the purpose of evaluating their role in the supply of soluble metals to soils.

Figure 1 shows that after a rapid initial rise, the dissolution rate of nearly all elements decreases rapidly. The dissolution capacity of the three humic acids is fairly similar. Ca, Mg, Co and Ni came to near equilibrium after 6–12 h, Al and Zn after 36 h. Fe, Mn, Cr and, to a lesser extent, Cu continue to be dissolved, though at a slow rate, after 108 h. Fe was the most extracted metal, followed by Al, Ca and Mg (Table 1). The transition metals were extracted in trace amounts. Relative to their contents in the basalt, Cu was the most extracted metal, followed by Zn. Next came Co, Cr and Mn, then Ni. Less than 0.5% of Fe, Al, Mg and Ca were extracted by the humic acids. Arranged according to the relative amounts extracted, the following sequence is obtained: Cu > Zn > Mn > Cr > Co > Ni > Al > Fe > Mg > Ca. The large amounts of metals extracted by the humic acids indicate the significant role these organic compounds may have in weathering and soil formation. An even more important aspect of that capacity is the role humic acids may play making micronutrients available (particularly copper and zinc) to plants growing on basalt-derived soils.

The capacity of humic acids to dissolve metals has been given as up to 682 mg g^{-1} (ref. 4). Thus, the total amounts